Du 10 décembre au 16 Décembre

<u>Exercice 141</u>

$$\begin{cases} -2x - 3y + 3z = 1 \\ x + 2y - z = 0 \\ x + y - 2z = 2. \end{cases}$$

$$\Leftrightarrow \begin{cases} x + y - 2z = 2 \\ x + 2y - z = 0 \\ -2x - 3y + 3z = 1 \end{cases} \qquad L_1 \leftrightarrow L_3.$$

$$\Leftrightarrow \begin{cases} x + y - 2z = 2 \\ x + 2y - z = 0 \\ 0 = 3. \end{cases} \qquad L_3 \leftarrow L_3 + L_1 + L_2.$$

Donc $\boxed{\mathcal{S} = \emptyset}$

<u>Exercice 142</u>

$$\begin{cases} y - z = 1 \\ 2x + y + 2z = 3 \\ x + z = 1 \end{cases}$$

$$\Leftrightarrow \begin{cases} x + z = 1 \\ y - z = 1 \\ 2x + y + 2z = 3 \end{cases} \qquad L_1 \to L_2 \to L_3 \to L_1$$

$$\Leftrightarrow \begin{cases} x + z = 1 \\ y - z = 1 \\ y = 1 \end{cases} \qquad L_3 \leftarrow L_3 - 2L_1.$$

$$\Leftrightarrow \begin{cases} x = +1 \\ z = 0 \\ y = 1 \end{cases}$$

$$\boxed{\mathcal{S} = \{(1,0,1)\}}$$

Exercice 143

$$\begin{cases} x + y + z + t = 0 \\ x - y - z = 0 \\ y + z - t = 0 \\ x - z + t = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x + y + z + t = 0 \\ 2y + 2z = 0 & L_2 \leftarrow L_1 - L_2. \\ y + z + t = 0 \\ y + 2z = 0 & L_4 \leftarrow L_1 - L_4 \end{cases}$$

$$\Leftrightarrow \begin{cases} x + y + z + t = 0 \\ y + z = 0 & L_2 \leftarrow L_2 \div 2 \\ y + z + t = 0 \\ y + 2z = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x + y + z + t = 0 \\ y + z = 0 \\ t = 0 & L_3 \leftarrow L_3 - L_2 \\ z = 0 & L_4 \leftarrow L_4 - L_2 \end{cases}$$

$$\Leftrightarrow \left\{ x = y = z = t = 0 \right. \qquad \boxed{\mathcal{S} = \{(0,0,0,0)\}}$$

Exercice 144

$$\begin{cases} x + y + z = a \\ x - y - z = b \\ -3x + y + 3z = c \end{cases}$$

$$\Leftrightarrow \begin{cases} x + y + z = a \\ 2y + 2z = a - b & L_2 \leftarrow L_1 - L_2 \\ 4y + 6z = c + 3a & L_3 \leftarrow L_3 + 3L_1 \end{cases}$$

$$\Leftrightarrow \begin{cases} x + y + z = a \\ 2y + 2z = a - b \\ 2z = c + 3a - 2a + 2b & L_3 \leftarrow L_3 - 2L_2. \\ = c + 2b + a \end{cases}$$

Ainsi

$$\boxed{z = \frac{1}{2}\left(c + 2b + a\right)}$$

$$2y = a - b - (c + 2b + a)$$
$$= -3b - c$$
$$\Leftrightarrow \boxed{y = -\frac{3}{2}b - \frac{c}{2}}$$

$$x = a - \left(-\frac{3}{2}b - \frac{c}{2}\right) - \frac{1}{2}(c + 2b + a)$$

$$\boxed{x = \frac{1}{2}a + \frac{1}{2}b}$$

Exercice 145

$$\left(\begin{array}{ccc|ccc} 3 & -2 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 3 & -2 & 0 & 1 & 0 & 0 \end{array}\right) \quad L_3 \leftrightarrow L_1$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 5 & 0 & -1 & 0 & 3 \end{array}\right) \quad \begin{array}{l} L_2 \leftarrow L_1 - L_2 \\ L_3 \leftarrow 3L_1 - L_3 \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 & 5 & -2 \end{array}\right) \quad L_3 \leftarrow L_3 - 5L_2$$

La matrice A n'est pas inversible.

Exercice 146

$$\left(\begin{array}{ccc|ccc} -1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} -1 & 0 & 2 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right) \quad L_3 \leftrightarrow L_2.$$

$$\left(\begin{array}{ccc|ccc} -1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right) \quad L_2 \leftarrow L_3 - L_2.$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 2 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right) \quad L_1 \leftarrow 2L_3 - L_1$$

La matrice C est inversible

et $\boxed{C^{-1} = \begin{pmatrix} -1 & 2 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 0 \end{pmatrix}}$

Exercice 14 :

$$\begin{cases} (1-\lambda)x - y + 2z = 0 \\ x - (1+\lambda)y + 2z = 0 \\ x - y + (2-\lambda)z = 0 \end{cases}$$

$$\iff \begin{cases} x - y + (2-\lambda)z = 0 \\ x - (1+\lambda)y + 2z = 0 \\ (1-\lambda)x - y + 2z = 0 \end{cases} \qquad L_3 \leftrightarrow L_1$$

$$\iff \begin{cases} x - y + (2-\lambda)z = 0 \\ -\lambda y + \lambda z = 0 \\ (1-\lambda)x - y + 2z = 0 \end{cases} \qquad L_2 \leftarrow L_2 - L_1$$

$$\iff \begin{cases} x - y + (2-\lambda)z = 0 \\ -\lambda y + \lambda z = 0 \\ -\lambda y + \lambda(3-\lambda)z = 0 \end{cases} \qquad L_3 \leftarrow L_3 - (1-\lambda)L_1$$

$-1 + (1-\lambda) = -\lambda$

$2 - (1-\lambda)(2-\lambda) = 2 - 2 + 3\lambda - \lambda^2 = 3\lambda - \lambda^2 = \lambda(3-\lambda)$

$$\iff \begin{cases} x - y + (2-\lambda)z = 0 \\ -\lambda y + \lambda z = 0 \\ \lambda(2-\lambda)z = 0 \end{cases} \qquad L_3 \leftarrow L_3 - L_2$$

$\lambda(3-\lambda) - \lambda = \lambda(2-\lambda)$

Le système est de Cramer lorsque $\lambda \in \mathbb{R} \setminus \{0;2\}$.

- Lorsque $\lambda \in \mathbb{R}\setminus\{0;2\}$: $\mathcal{S} = \{(0,0,0)\}$ (le système est de Cramer).

- Lorsque $\lambda = 0$

Le système s'écrit :

$$\begin{cases} x - y + 2z = 0 \\ 0 = 0 \\ 0 = 0 \end{cases}$$

$$\iff \begin{cases} x = y - 2z \\ 0 = 0 \\ 0 = 0 \end{cases}$$

$$\mathcal{S} = \{(y-2z, y, z),\ (y,z) \in \mathbb{R}^2\}.$$

Lorsque $\lambda = 2$.

Le système s'écrit

$$\begin{cases} x - y = 0 \\ -2y + 2z = 0 \\ 0 = 0 \end{cases}$$

$$\iff \begin{cases} x = -z \\ y = -z \\ 0 = 0 \end{cases}$$

$$\mathcal{S} = \{(-z, -z, z),\ z \in \mathbb{R}\}.$$